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IMPROVING ANGULAR ACCURACY BY THRESHOLDING.(U)

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IMPROVING ANGULAR ACCURACY BY THRESHOLDING

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AUGUST 1981

U.S. ARMY MISSILE COMMAND
Redstone Arsenal, Alabama 35898

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This paper presents the anlaysis of a signal processor that senses the phase front distortion across a monopulse antenna due to glint phenomena and rejects the consequent erroneous tracking data. The resulting glint reduction achieved by applying a combination of thresholds to the normalized sum channel signal and to the normalized imaginary component of the complex ratio D/S is presented. The results show that for a specified fraction of rejected data, maximum glint reduction is obtained by thresholding on sum channel information alone.

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I. INTRODUCTION AND SUMMARY

As a homing missile closes on its target, a larger portion of its seeker beam is subtended by the finite angular extent of the target. The radar echoes from the target no longer appear to arrive from a single direction. The direction to the target is determined from the measured amplitudes and phases of the received difference and sum channel signals as $\frac{|D|}{|S|} \cos (\phi_D - \phi_S)$.

Mutual interference between the echoes from individual scatters causes the phase front across the antenna (and hence the apparent direction to the target) to oscillate about its mean. For a point target the phasors D and S are colinear. When phase front distortion occurs, the quantity $\frac{|D|}{|S|} \sin (\phi_D - \phi_S)$ is no longer zero and the sum channel amplitude undergoes a fade. The apparent glint in the resulting target direction can be reduced by designing a signal processor which senses the occurrence of this interference phenomenon and rejects the consequent erroneous tracking data.

This paper presents the analysis of such a processor and shows the glint reduction achieved by applying a combination of thresholds to $|S|$ and $\frac{|D|}{|S|} |\sin (\phi_D - \phi_S)|$. Curves are presented which show the fraction of data rejected by the thresholding procedure as a function of the two applied thresholds. Two measures of glint reduction are evaluated; curves are presented for each of these measures which show the glint reduction obtained versus the fraction of data lost. For a specified fraction of rejected data, maximum glint reduction is obtained by thresholding on $|S|$ alone.

II. ANALYSIS

The homing missile employs a phase comparison monopulse seeker; each subaperture of the antenna is assumed to possess an isotropic gain pattern. We consider the glint effects in a single angle dimension only (Figure 1).

If a monochromatic wave is reflected from a point target at range R>>d and off-boresight angle ψ then the RF signals received at $-\frac{d}{2}$, $+\frac{d}{2}$ respectively are given by

$$V\left(-\frac{d}{2}\right) = \frac{1}{4\pi \left(R + \frac{d}{2} \sin \psi\right)} A' e^{i2\pi f \left(t - \frac{R + \frac{d}{2} \sin \psi}{c}\right)} \quad (1)$$

$$V\left(+\frac{d}{2}\right) = \frac{1}{4\pi \left(R - \frac{d}{2} \sin \psi\right)} A' e^{i2\pi f \left(t - \frac{R - \frac{d}{2} \sin \psi}{c}\right)} \quad (2)$$

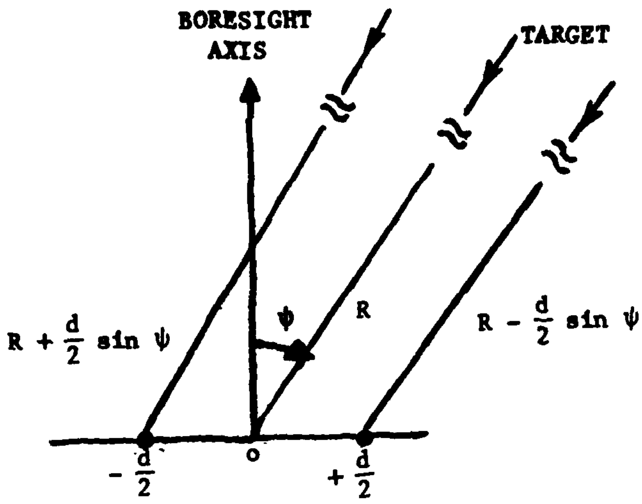

Figure 1. Two phase centers of seeker antenna model.

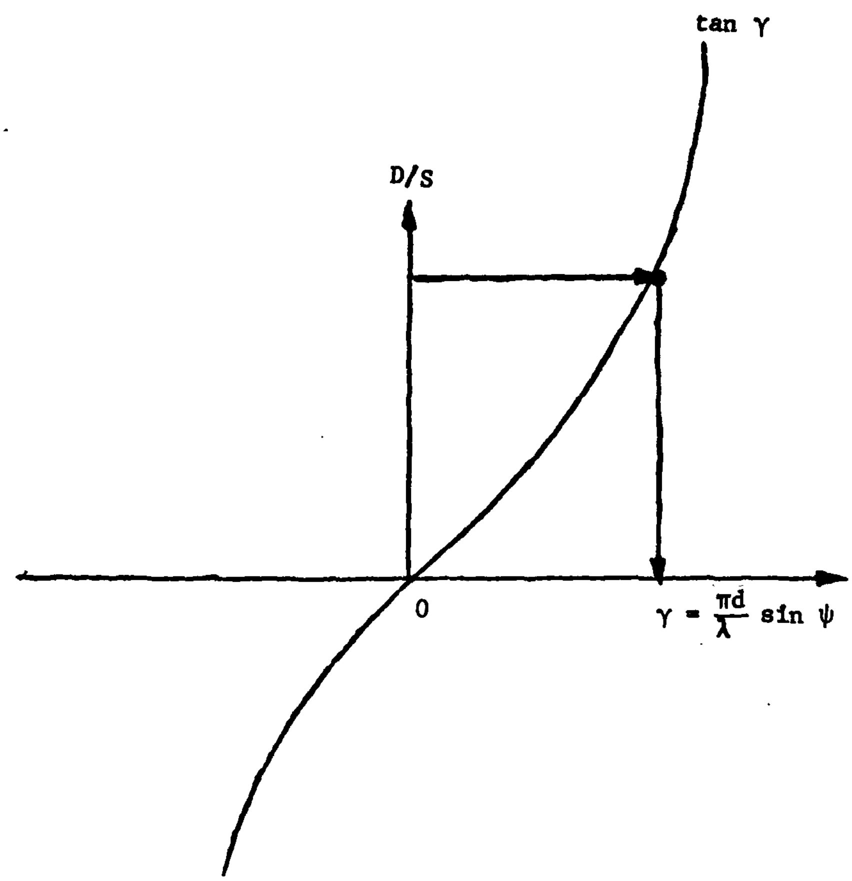

Figure 2. Monopulse error characteristic.

where A´ contains the target amplitude and target phase referenced to the transmitted signal from the illuminating radar.

Magic T's at the antenna combine these signals so that the receiver input consists of the sum and difference signals.

$$S \triangleq V\left(-\frac{d}{2}\right) + V\left(+\frac{d}{2}\right) \tag{3}$$

$$D \triangleq +i\left[V\left(-\frac{d}{2}\right) - V\left(+\frac{d}{2}\right)\right] \tag{4}$$

Since R>>d, the complex envelopes of the IF signals in the sum and difference channels of the receiver may be written as

$$S = A \cos \gamma \triangleq u + iv \tag{5}$$

$$D = A \sin \gamma \triangleq X + iy \tag{6}$$

where the complex amplitude A includes the effect of range attenuation, target RF phase, target cross-section, etc., and we have denoted by

$$\gamma(\psi) = \frac{\pi d}{\lambda} \sin \psi \tag{7}$$

the RF phase shift for a target at ψ referred to the RF phase which would be received at the antenna center. (For an active missile γ is replaced throughout by 2 γ). For a single target the ratio D/S is real and is independent of the target properties, A. Thus by forming D/S the receiver obtains tan γ from which the off-boresight angle ψ is uniquely determined (Figure 2).

Although for a point target

$$\frac{D}{S} = \frac{x}{u} = \frac{y}{v} \text{ (real)} \tag{8}$$

receivers are designed to produce the output

$$\left|\frac{D}{S}\right| \cos\,(\phi_D - \phi_S) = \frac{xu + yv}{u^2 + v^2} = \text{Re}\ (D/S) \tag{9}$$

This has the virtue of not becoming indeterminate when either the phase of A is 0 or π ($y=v=0$) or the phase of A is $\frac{\pi}{2}$ or $\frac{3\pi}{2}$ ($x=u=0$); for a point target Re D/S gives the same monopulse error characteristic as does D/S.

Consider now signals returned from an extended target which occupies the segment of the beam from $\psi_o - \frac{\delta}{2}$ to $\psi_o + \frac{\delta}{2}$. The complex envelopes in the sum and difference channels of the receiver may be written as

$$S = \int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} A(\psi)\ \cos\ \gamma(\psi)\ d\psi + n_S \triangleq u + iv \tag{10}$$

$$D = \int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} A(\psi)\ \sin\ \gamma(\psi)\ d\psi + n_D \triangleq x + iy \tag{11}$$

where n_S and n_D are independent receiver noises in the two channels.

Denote statistical averages over the noise, the amplitudes of the individual scatterers and the phases of the individual scatterers by an overbar. Then assuming that the phases of the individual scatterers are independent and uniformly distributed,

$$\overline{S} = \overline{u} + i\overline{v} = 0 \tag{12}$$

$$\overline{D} = \overline{x} + i\overline{y} = 0 \tag{13}$$

Since x and u involve only the cosines of the target phases while y and v involve only the sines there is zero correlation between any in-phase and any quadrature signals

$$\overline{xy} = \overline{uv} = \overline{xv} = \overline{yu} = 0 \tag{14}$$

Further, for either of the channels the average in-phase and quadrature powers are equal

$$\overline{u^2} = \overline{v^2} = \frac{1}{2} \int_{\psi_0-\frac{\delta}{2}}^{\psi_0+\frac{\delta}{2}} \overline{|A(\psi)|^2} \cos^2\gamma(\psi)d\psi + \frac{1}{2}\overline{|n_S|^2} \triangleq b^2 \tag{15}$$

$$\overline{x^2} = \overline{y^2} = \frac{1}{2} \int_{\psi_0-\frac{\delta}{2}}^{\psi_0+\frac{\delta}{2}} \overline{|A(\psi)|^2} \sin^2\gamma(\psi)d\psi + \frac{1}{2}\overline{|n_D|^2} \triangleq a^2 \tag{16}$$

Also, the average cross-channel powers are equal

$$\overline{xu} = \overline{yv} = \frac{1}{2} \int_{\psi_0-\frac{\delta}{2}}^{\psi_0+\frac{\delta}{2}} \overline{|A(\psi)|^2} \sin\gamma(\psi) \cos\gamma(\psi)\, d\psi \triangleq \rho ab \tag{17}$$

Thus $2b^2$, $2a^2$ and $2\rho ab$ denote respectively the average sum channel, difference channel and cross-channel powers.

The extended target is assumed to consist of a large number of mutually interfering scatterers whose returns generate a Gaussian process.

Thus, (12) ..... (17) imply that the in-phase pairs x, u and the quadrature pairs y, v obey independent, bivariate, zero-mean Gaussian distributions. Because of this independence the joint probability density function (pdf) of x, y, u, v is given by

$$p_{x,y,u,v}(x,y,u,v) = p_{x,u}(x,u)\, p_{y,v}(y,v) \tag{18}$$

where

$$p_{x,u}(x,u) = \frac{1}{2\pi ab\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{x^2}{a^2} - \frac{2\rho xu}{ab} + \frac{u^2}{b^2}\right]} \tag{19}$$

$$p_{y,v}(y,v) = \frac{1}{2\pi ab\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{y^2}{a^2} - \frac{2\rho yv}{ab} + \frac{v^2}{b^2}\right]} \tag{20}$$

Introduce

$$\mu \triangleq \frac{a}{b}\sqrt{1-\rho^2} = \frac{\left[\overline{|D|^2}\ \overline{|S|^2} - \overline{|DS^*|}^2\right]^{1/2}}{\overline{|S|^2}^2} \tag{21}$$

Then (18) yields

$$p_{x,y,u,v}(x,y,u,v) = \frac{1}{(2\pi)^2 b^4 \mu^2} e^{-\frac{1}{2b^2\mu^2}\left[(x^2 + y^2) - \frac{2\rho a}{b}(xu + yv) + \frac{a^2}{b^2}(u^2 + v^2)\right]} \tag{22}$$

In order to analyze the glint from the extended target we require the statistical properties of the sum channel amplitude and the real and imaginary parts of the complex monopulse ratio. Thus we introduce the one-to-one transformation of variables

$$\xi \triangleq \frac{xu + yv}{u^2 + v^2} = \mathrm{Re}(D/S) \quad (-\infty, \infty) \tag{23}$$

$$\eta \triangleq \frac{yu - xv}{u^2 + v^2} = \mathrm{Im}(D/S) \quad (-\infty, \infty) \tag{24}$$

$$\zeta \triangleq \sqrt{u^2 + v^2} = |S| \; (0,\infty) \tag{25}$$

$$\theta \triangleq \tan^{-1} \frac{v}{u} = \text{phase of } S \; (0,2\pi) \tag{26}$$

The pdf for the transformed variables may be determined from the transformation rule

$$p_{\xi,\eta,\zeta,\theta} (\xi,\eta,\zeta,\theta) = p_{x,y,u,v} \left[x (\xi,\eta,\zeta,\theta), y(\xi,\eta,\zeta,\theta), u(\xi,\eta,\zeta,\theta), v(\xi,\eta,\zeta,\theta)\right] J \tag{27}$$

where the Jacobian of the transformation is given by the determinant

$$J = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial x}{\partial \eta} & \frac{\partial x}{\partial \zeta} & \frac{\partial x}{\partial \theta} \\ - & - & - & - \\ \frac{\partial v}{\partial \xi} & \frac{\partial v}{\partial \eta} & \frac{\partial v}{\partial \zeta} & \frac{\partial v}{\partial \theta} \end{bmatrix} \tag{28}$$

The inverse transformation is given by

$$x = \zeta(\xi \cos \theta - \eta \sin \theta) \tag{29}$$

$$y = \zeta(\xi \sin \theta + \eta \cos \theta) \tag{30}$$

$$u = \zeta \cos \theta \tag{31}$$

$$v = \zeta \sin \theta \tag{32}$$

so that the Jacobian is explicitly given by

$$J = \begin{bmatrix} \zeta \cos \theta & -\zeta \sin \theta & \xi \cos \theta - \eta \sin \theta & \zeta(\xi \sin \theta + \eta \cos \theta) \\ \zeta \sin \theta & \zeta \cos \theta & \xi \sin \theta + \eta \cos \theta & \zeta(\xi \cos \theta - \eta \sin \theta) \\ 0 & 0 & \cos \theta & -\zeta \sin \theta \\ 0 & 0 & \sin \theta & \zeta \cos \theta \end{bmatrix} = \zeta^3 \tag{33}$$

The joint pdf for the transformed variables becomes

$$p_{\xi,\eta,\zeta,\theta}(\xi,\eta,\zeta,\theta) = \frac{\zeta^3}{(2\pi)^2 b^4 \mu^2} e^{-\frac{\zeta^2}{2b^2\mu^2}\left[\xi^2+\eta^2 -2\frac{\rho a}{b}\xi+\frac{a^2}{b^2}\right]} \tag{34}$$

Since θ is uniformly distributed in $(0,2\pi)$, we have the desired joint pdf for the variables of interest

$$p_{\xi,\eta,\zeta}(\xi,\eta,\zeta) = \frac{\zeta^3}{2\pi b^4 \mu^2} e^{-\frac{\zeta^2}{2b^2}\left[\left(\frac{\xi-\frac{\rho a}{b}}{\mu}\right)^2 + \left(\frac{\eta}{\mu}\right)^2 + 1\right]} \tag{35}$$

From (35) it immediately follows that the pdf's for the variations of ξ and η about their means are identical, i.e.,

$$p_\xi(\xi+\frac{\rho a}{b}) = p_\eta(\xi) \tag{36}$$

that each pdf is unimodal and symmetric about its mean, and that these means are

$$\overline{\xi} = \frac{\rho a}{b} \tag{37}$$

$$\overline{\eta} = 0 \tag{38}$$

Introduce the variables

$$s \triangleq \frac{\xi - \frac{\rho a}{b}}{\mu} \quad \text{normalized Re (D/S)} \tag{39}$$

$$t \triangleq \frac{\eta}{\mu} \quad \text{normalized Im (D/S)} \tag{40}$$

$$z \triangleq \frac{\zeta}{\sqrt{2}b} \quad \text{normalized } |S| \tag{41}$$

which represent respectively the deviations of ξ and η from their means normalized to μ and the sum channel amplitude normalized to the average sum channel amplitude. Since the Jacobian of the transformation is equal to $\frac{1}{\sqrt{2}b\mu^2}$, the joint pdf for the variables s, t, z is given by

$$p_{s,t,z}(s,t,z) = \frac{2}{\pi} z^3 e^{-z^2(s^2+t^2+1)} \tag{42}$$

The variables s, t are not independent; their joint pdf is given by

$$p_{s,t}(s,t) = \int_0^\infty \frac{2z^3}{\pi} e^{-z^2(s^2+t^2+1)} dz = \frac{1}{\pi(s^2+t^2+1)^2} \tag{43}$$

Hence the pdf of s is finally obtained as

$$p_s(s) = \int_{-\infty}^{\infty} \frac{1}{\pi(s^2+t^2+1)^2} dt = \frac{1}{(s^2+1)^{3/2}} \frac{1}{\pi} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sec^2\phi}{\sec^4\phi} d\phi = \frac{1}{2(s^2+1)^{3/2}} \tag{44}$$

The pdf of Re(D/S) is given by [cf (39)].

$$p_\xi(\xi) = \frac{1}{2\mu} \frac{1}{\left[\left(\frac{\xi-\bar{\xi}}{\mu}\right)^2 +1\right]^{3/2}} \qquad \left(\sigma_\xi^2 = \infty\right) \tag{45}$$

and the pdf of Im(D/S) is given by [cf (36)].

$$p_\eta(\eta) = \frac{1}{2\mu} \frac{1}{\left[\left(\frac{\eta}{\mu}\right)^2 +1\right]^{3/2}} \qquad \left(\sigma_\eta^2 = \infty\right) \tag{46}$$

Since the tails of these pdf's do not decay faster than ξ^{-3} or η^{-3} their second moments are infinite; as a measure of spread about the mean we choose the first absolute central moment, i.e.,

$$\overline{|\xi-\bar{\xi}|} = \frac{1}{2\mu}\int_{-\infty}^{\infty} \frac{|\xi-\bar{\xi}|\,d\xi}{\left[\left(\frac{\xi-\bar{\xi}}{\mu}\right)^2+1\right]^{3/2}} = \mu \qquad (47)$$

We know that for a Gaussian pdf the variance, σ_G^2, is finite and the first absolute central moment is directly proportional to the standard deviation, i.e.,

$$\mu_G = \int_{-\infty}^{\infty} |x-\bar{x}| \frac{1}{\sqrt{2\pi}\sigma_G} e^{\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma_G}\right)^2} dx = \sqrt{\frac{2}{\pi}}\,\sigma_G \qquad (48)$$

Thus the normalizing parameter, μ, defined by (21) corresponds to a standard deviation.

We next consider how well thresholding on t and z (or η and ζ) reduces the spread (measured by μ) of the pdf of s (or ξ).

The pdf of s conditioned on an arbitrary event A is

$$p_{s|A}(s|A) = \frac{p(s,A)}{\text{Prob. }(A)} \qquad (49)$$

Let us now consider two thresholds T and Z and a processor which accepts measurements of $R_e(D/S)$ only when $|Im(D/S)|$ is sufficiently small and $|S|$ is sufficiently large. In (49), the compound event A is taken to describe the set of outcomes

$$A \triangleq \{t,z: |t| < T, z>Z\} \qquad (50)$$

so that (49) becomes

$$p_{s|A}\left(s \,\middle|\, |t|<T,\ z>Z\right) = \frac{\int_Z^{\infty}\int_{-T}^{T} p(s,t,z)\,dt\,dz}{\int_{-\infty}^{\infty}\int_Z^{\infty}\int_{-T}^{T} p(s,t,z)\,dt\,dz\,ds} \triangleq \frac{G(s,T,Z)}{H(T,Z)} \tag{51}$$

Employing (42) we have

$$G(s,T,Z) = \int_Z^{\infty} \frac{2}{\pi} z^3 e^{-z^2(s^2+1)} \left[2\int_0^T e^{-z^2t^2}dt\right] dz \tag{52}$$

In terms of the error function defined by

$$\text{erf } x \triangleq \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt,\ \text{erf}(\infty) = 1 \tag{53}$$

the numerator of (51) is given by

$$G(s,T,Z) \triangleq \frac{2}{\sqrt{\pi}}\int_Z^{\infty} z^2 e^{-z^2(s^2+1)} \text{ erf }(Tz)dz \tag{54}$$

The denominator, representing the fraction of data which is accepted, is

$$H(T,Z) \triangleq \int_{-\infty}^{\infty} G(s,T,Z)ds = \int_Z^{\infty} 2ze^{-Z^2} \text{ erf }(Tz)dz \tag{55}$$

Integration by parts yields

$$H(T,Z) = \left[-e^{-z^2} \text{ erf } Tz\right]_Z^{\infty} + T\frac{2}{\sqrt{\pi}}\int_Z^{\infty} e^{-z^2(T^2+1)}dz \tag{56}$$

In terms of the complementary error function defined by

$$\operatorname{erfc} x \triangleq 1-\operatorname{erf} x = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \tag{57}$$

The denominator of (51) becomes

$$H(T,Z) = e^{-Z^2} \operatorname{erf}(TZ) + \frac{1}{\sqrt{T^2+1}} \operatorname{erfc}\left(Z\sqrt{T^2+1}\right) \tag{58}$$

Thus the pdf of s conditioned on the event $\{t,z: |t|<T, z>Z\}$ is

$$p_{s|A}(s\,|\,|t|<T,z>Z) = \frac{G(s,T,Z)}{H(T,Z)} = \frac{\frac{2}{\sqrt{\pi}} \int_{Z}^{\infty} z^2 e^{-z^2(s^2+1)} \operatorname{erf}(Tz)\,dz}{e^{-Z^2} \operatorname{erf}(TZ) + \frac{T}{\sqrt{T^2+1}} \operatorname{erfc}(Z\sqrt{T^2+1})} \tag{59}$$

This conditional pdf has a mean of zero and a smaller deviation about its mean than does the unconditional pdf given by (44). Two measures of this deviation will be considered: the conditioned first absolute central moment defined by

$$\mu_c(T,Z) \triangleq \int_{-\infty}^{\infty} |s|\, p_{s|A}\left(s \,\middle|\, |t|<T,z>Z\right) ds \triangleq \frac{G_1(T,Z)}{H(T,Z)} \tag{60}$$

and the conditioned standard deviation defined by

$$\sigma_c(T,Z) \triangleq \left[\int_{-\infty}^{\infty} s^2\, p_{s|A}\left(s \,\middle|\, |t|<T,z>Z\right) ds\right]^{\frac{1}{2}} = \left[\frac{G_2(T,Z)}{H(T,Z)}\right]^{\frac{1}{2}} \tag{61}$$

where

$$G_1(T,Z) = \int_{-\infty}^{\infty} |s| G(s,T,Z)ds = \frac{2}{\sqrt{\pi}} \int_Z^{\infty} e^{-z^2} \operatorname{erf}(Tz)dz \tag{62}$$

$$G_2(T,Z) = \int_{-\infty}^{\infty} s^2 G(s,T,Z)ds = \int_Z^{\infty} \frac{1}{z} e^{-z^2} \operatorname{erf}(Tz)dz \tag{63}$$

Recalling the relation (39) between s and ξ, we note that both the conditioned first absolute central moment and the conditioned standard deviation of the pdf of ξ are directly proportional to μ. Thus the glint reduction in Re D/S accomplished by thresholding will be defined by the glint reduction factors

$$\frac{\mu}{\mu_c(T,Z)} \triangleq \frac{H(T,Z)}{G_1(T,Z)} \tag{64}$$

$$\frac{\mu}{\sqrt{\frac{2}{\pi}}\sigma_c(T,Z)} \triangleq \left[\frac{\pi}{2}\,\frac{H(T,Z)}{G_2(T,Z)}\right]^{\frac{1}{2}} \tag{65}$$

where $H(T,Z)$, $G_1(T,Z)$, $G_2(T,Z)$ are given by (58), (62), (63) respectively.

We next solve the following problems: choose the thresholds T,Z so as to maximize the two glint reduction factors subject to a specified fraction of data rejected. (These isoperimetric problems are equivalent to the problems: choose the thresholds so as to minimize the fraction of data rejected while achieving the specified glint reduction factors).

Suppose we choose to maximize $\frac{\mu}{\mu_c(T,Z)}$ subject to the constraint

$$H(T,Z) = H_o \tag{66}$$

If we could eliminate T from (66) we would have a minimization problem in Z alone, i.e., we would seek the Z which satisfies

$$\frac{d}{dZ} \frac{H\,(T(Z),Z)}{G_1(T(Z),Z)} = 0 \tag{67}$$

Unfortunately an explicit expression for T(Z) is unattainable. However, since (66) and (67) are equivalent respectively to

$$\frac{\partial H}{\partial T}\frac{dT}{dZ} + \frac{\partial H}{\partial Z} = 0 \tag{68}$$

$$\left(G_1\frac{\partial H}{\partial T} - H\frac{\partial G_1}{\partial T}\right)\frac{dT}{dZ} + \left(G_1\frac{\partial H}{\partial Z} - H\frac{\partial G_1}{\partial Z}\right) = 0 \tag{69}$$

we eliminate $\frac{dT}{dZ}$ and observe that the extremizing thresholds T,Z must satisfy the resulting equation

$$\frac{\partial H}{\partial T}\frac{\partial G_1}{\partial Z} - \frac{\partial H}{\partial Z}\frac{\partial G_1}{\partial T} = 0 \tag{70}$$

Using (55) and (62) the required partial derivatives are given by

$$\frac{\partial H}{\partial T} = \frac{Z}{T^2+1}\left[\frac{2}{\sqrt{\pi}}e^{-Z^2(T^2+1)} + \frac{1}{Z\sqrt{T^2+1}}\operatorname{erfc}(Z\sqrt{T^2+1})\right] \tag{71}$$

$$\frac{\partial H}{\partial Z} = -2Ze^{-Z^2}\operatorname{erf}(TZ) \tag{72}$$

$$\frac{\partial G_1}{\partial Z} = \frac{-2}{\sqrt{\pi}}e^{-Z^2}\operatorname{erf}(TZ) \tag{73}$$

$$\frac{\partial G_1}{\partial T} = \frac{2}{\pi} \frac{1}{T^2+1} e^{-Z^2(T^2+1)} \tag{74}$$

When these are employed in (70) we obtain

$$\frac{e^{-Z^2}}{(T^2+1)^{3/2}} \operatorname{erf}(TZ) \operatorname{erfc}(Z\sqrt{T^2+1}) = 0 \tag{75}$$

Each of the two solutions, $T=\infty$ and $Z=0$, extremize the glint reduction factor (64) subject to the constraint (66). In particular when we threshold only on the sum channel amplitude ($T=\infty$) we have from (58), (62)

$$H(\infty, Z) = e^{-Z^2} = H_o \tag{76}$$

$$G_1(\infty, Z) = \operatorname{erfc} Z \tag{77}$$

Solving (76) for the threshold, Z, in terms of the fraction of data retained we have

$$Z = (\ln\frac{1}{H_o})^{1/2} \tag{78}$$

Thus one extremal of the glint reduction factor is expressed in terms of the fraction of data retained by

$$\frac{\mu}{\mu_c(\infty, Z)} = \frac{H(\infty, Z)}{G_1(\infty, Z)} = \frac{H_o}{\operatorname{erfc}\left[\left(\ln\frac{1}{H_o}\right)^{1/2}\right]} \tag{79}$$

The other extremal solution is obtained by thresholding only on the imaginary part of D/S, i.e., Z=0. For this situation (58) and (62) yield respectively

$$H(T,0) = \frac{T}{\sqrt{T^2+1}} = Ho \tag{80}$$

$$G_1(T,0) = \frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-z^2} \operatorname{erf}(Tz)dz \tag{81}$$

The integral may be evaluated by noting that

$$\frac{dG_1}{dT}(T,0) = \frac{2}{\pi} \int_0^{\infty} 2ze^{-z^2(T^2+1)}dz = \frac{2}{\pi} \frac{1}{T^2+1} \tag{82}$$

Since

$$G_1(0,0) = 0 \tag{83}$$

we have

$$G_1(T,0) = \int_0^T \frac{2}{\pi} \frac{1}{z^2+1} dz = \frac{2}{\pi} \tan^{-1} T \tag{84}$$

From (80) we find the threshold in terms of the fraction of data retained

$$T = \frac{Ho}{\sqrt{1-Ho^2}} \tag{85}$$

hence the second extremal of the glint reduction factor expressed in terms of the fraction of data retained is given by

$$\frac{\mu}{\mu_c(T,0)} = \frac{H(T,0)}{G_1(T,0)} = \frac{Ho}{\frac{2}{\pi} \tan^{-1}\left(\frac{Ho}{\sqrt{1-Ho^2}}\right)} \tag{86}$$

For any other combinations of thresholds which satisfy the constraint

$$e^{-Z^2}\,\mathrm{erf}(TZ) + \frac{T}{\sqrt{T^2+1}}\,\mathrm{erfc}(Z\sqrt{T^2+1}) = Ho \tag{87}$$

we obtain the glint reduction factor

$$\frac{\mu}{\mu_c(T,Z)} = \frac{H\ (T,Z)}{G_1(T,Z)} = \frac{Ho}{\frac{2}{\sqrt{\pi}}\int_Z^\infty e^{-z^2}\,\mathrm{erf}(Tz)\,dz} \tag{88}$$

which is intermediate between the extremal values given by (79) and (86).

The second measure of glint reduction is given by (65). Using (63) we find the partial derivatives.

$$\frac{\partial G_2}{\partial Z} = \frac{-1}{Z}e^{-Z^2}\,\mathrm{erf}(TZ) \tag{89}$$

$$\frac{\partial G_2}{\partial T} = \frac{1}{\sqrt{T^2+1}}\,\mathrm{erfc}(Z\sqrt{T^2+1}) \tag{90}$$

The equation for the extremizing thresholds,

$$\frac{\partial H}{\partial T}\frac{\partial G_2}{\partial Z} - \frac{\partial H}{\partial Z}\frac{\partial G_2}{\partial T} = 0 \tag{91}$$

thus yields

$$\frac{e^{-Z^2}\,\mathrm{erf}(TZ)}{Z(T^2+1)^{3/2}}\left\{\left[1-2Z^2(T^2+1)\right]\ \mathrm{erfc}(Z\sqrt{T^2+1}) + \frac{2}{\sqrt{\pi}}\,Z\sqrt{T^2+1}\ e^{-Z^2(T^2+1)}\right\} = 0 \tag{92}$$

Clearly one solution is given by $T=\infty$, i.e., threshold on only the sum channel amplitude. For this situation (58) becomes

$$H(\infty,Z) = e^{-Z^2} = Ho \tag{93}$$

and (63) becomes

$$G_2(\infty,Z) = \int_Z^\infty \frac{1}{z} e^{-z^2} dz = \frac{1}{2}\int_{Z^2}^\infty \frac{e^{-t}}{t} dt \triangleq \frac{1}{2} E_1(Z^2) \tag{94}$$

where $E_1(.)$ is the exponential integral. Note that this becomes unbounded as $Z \to 0$. This is to be expected since the situation $T=\infty$, $Z=0$ corresponds to $Ho=1$, i.e., to a processor which accepts all measurements of Re D/S. The pdf of its output is given by (45) which as we have already noted has infinite variance. From (93)

$$Z = (\ln\frac{1}{Ho})^{1/2} \tag{95}$$

so that an extremal of the second glint reduction factor is

$$\frac{\mu}{\sqrt{\frac{2}{\pi}}\sigma_c(\infty,Z)} = \left[\frac{\pi}{2}\frac{H(\infty,Z)}{G_2(\infty,Z)}\right]^{1/2} = \left[\frac{\pi Ho}{E_1(\ln\frac{1}{1-Ho})}\right]^{1/2} \tag{96}$$

We now prove that this is the only extremal. Note that any other combination of thresholds which provide an extremal must satisfy [cf.(92)],

$$(1-2x^2)\ \mathrm{erfc}(x) + \frac{2}{\sqrt{\pi}} x e^{-x^2} = 0 \tag{97}$$

where

$$x \triangleq Z\sqrt{T^2+1} > 0 \tag{98}$$

Clearly $x = \frac{1}{\sqrt{2}}$ is not a solution. Hence we seek values of x for which

$$f(x) \triangleq \int_x^\infty e^{-t^2} dt + \frac{xe^{-x^2}}{1-2x^2} = 0 \qquad (99)$$

Since

$$f(o) = \frac{\sqrt{\pi}}{2} > 0 \qquad (100)$$

and

$$f'(x) = \frac{4x^2 e^{-x^2}}{(1-2x^2)^2} > 0 \qquad (101)$$

we conclude that f(x) never vanishes for positive x and hence that (97) has no positive roots.

Thus the only extremal of $\frac{\mu}{\sqrt{\frac{2}{\pi}}\sigma_c(T,Z)}$ subject to the constraint (66) is given by (96). When we threshold on T alone we do not obtain an extremal, however the glint reduction factor for this case is obtained by noting that (58) and (63) yield respectively

$$H(T,0) = \frac{T}{\sqrt{T^2+1}} = Ho \qquad (102)$$

$$G_2(T,0) = \int_0^\infty \frac{1}{z} e^{-z^2} \operatorname{erf}(Tz)dz \qquad (103)$$

To evaluate the integral we note that

$$G_2(0,0) = 0 \qquad (104)$$

$$\frac{dG_2}{dT}(T,0) = \frac{2}{\sqrt{\pi}} \int_0^\infty e^{-z^2(T^2+1)} dz = \frac{1}{\sqrt{T^2+1}} \qquad (105)$$

hence

$$G_2(T,0) = \int_0^T \frac{1}{\sqrt{z^2+1}} dz = \int_0^{\tan^{-1}T} \sec\phi d\phi = \ln(T+\sqrt{T^2+1}) \qquad (106)$$

Note that $G_2(T,0)$ and hence $\sigma_c(T,0)$ has a logarithmic singularity at $T=\infty$. Using (102) this is given in terms of the fraction of data retained as

$$G_2(T,0) = \frac{1}{2} \ln \left(\frac{1+Ho}{1-Ho}\right) \qquad (107)$$

so that finally the (non-extremal) glint reduction factor obtained by thresholding on T alone is given by

$$\frac{\mu}{\sqrt{\frac{2}{\pi}}\sigma_c(T,0)} = \left[\frac{\pi}{2} \; \frac{H(T,0)}{G_2(T,0)}\right]^{1/2} = \left[\frac{\pi Ho}{\ln\left(\frac{1+Ho}{1-Ho}\right)}\right]^{1/2} \qquad (108)$$

III. DISCUSSION OF RESULTS

The results of this analysis are shown numerically in the accompanying graphs. Figures 3 and 4 show respectively graphs of the first glint reduction factor $\frac{\mu}{\mu_c(T,Z)}$ and the fraction of the data rejected $1-H(T,Z)$ vs sum channel amplitude threshold Z for values of fixed thresholds on $|Im\frac{D}{S}|$ equal to .05, .1, .3, .5, 1, 1.5, 2, 1000. Figure 5 shows the cross plots of $\frac{\mu}{\mu_c(T,Z)}$ and includes the two extremals. It is seen that for a given fraction of rejected data, maximum glint reduction is achieved by thresholding on Z alone. Along each of the intermediate curves T is held constant and Z(T) is chosen to yield

the specified value of 1-Ho. Figure 6 shows two examples of the second glint reduction factor vs 1-Ho. The upper curve is the sole extremal; the lower curve is a special threshold selection (z=0) which corresponds to thresholding only on $|Im(D/S)|$. As for the first glint reduction factor, thresholding on Z alone provides better performance.

The dashed curve in Figure 6 is a plot of the function $H^{-1/2}$ vs 1-H. It pertains to independent samples from any stationary random process with finite variance. It shows the reduction in the standard deviation of the sample mean if all samples are retained. This potential reduction is never realized when the process is Re D/S since the unthresholded samples do not have finite variance. Nevertheless we conclude that for almost the entire domain of the graph, we achieve a greater measure of glint reduction by retaining only the fraction H of the measurements whose sum channel amplitude exceeds the threshold Z than by forming a sample mean from all the independent measurements.

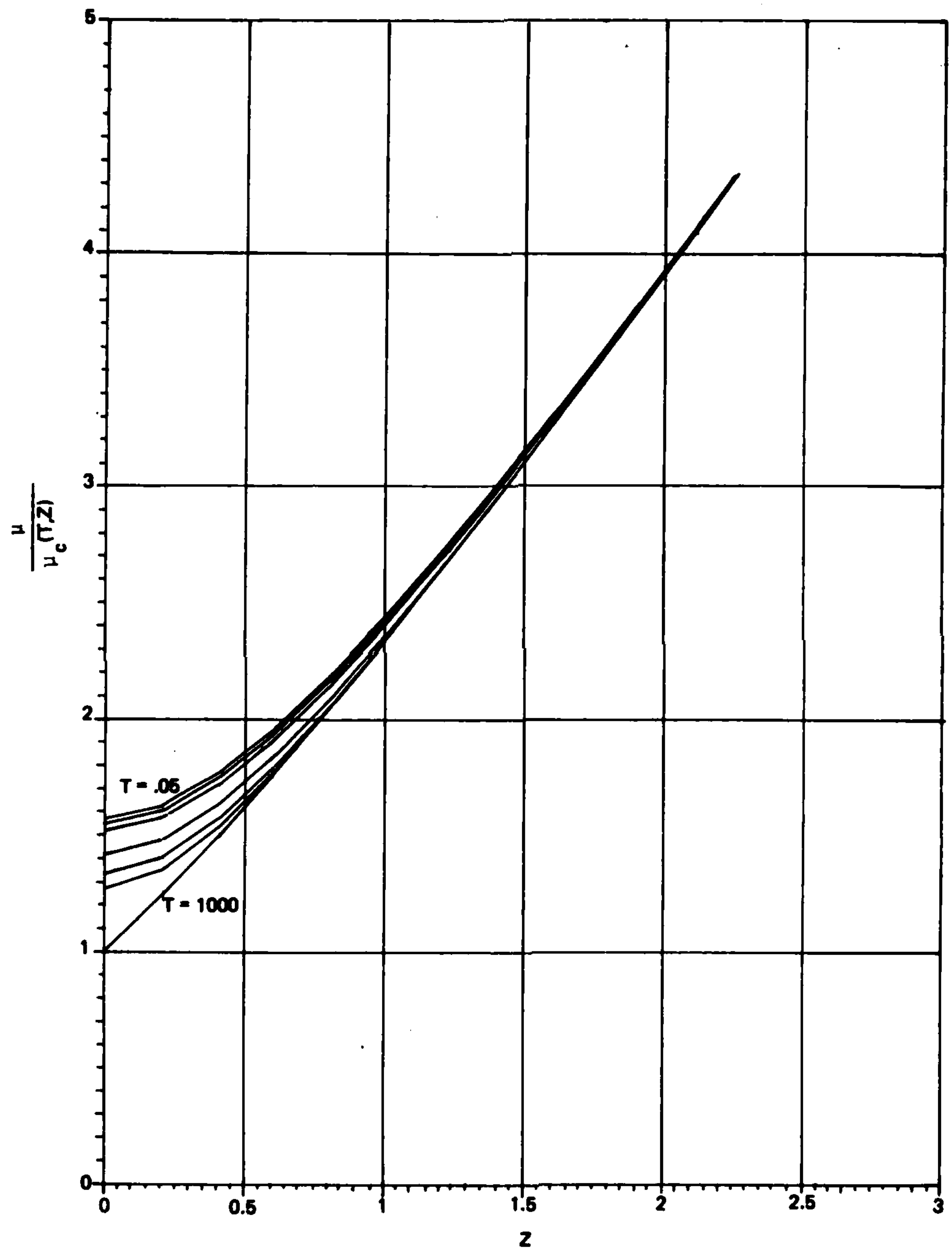

Figure 3. Glint reduction factor vs. sum channel threshold.

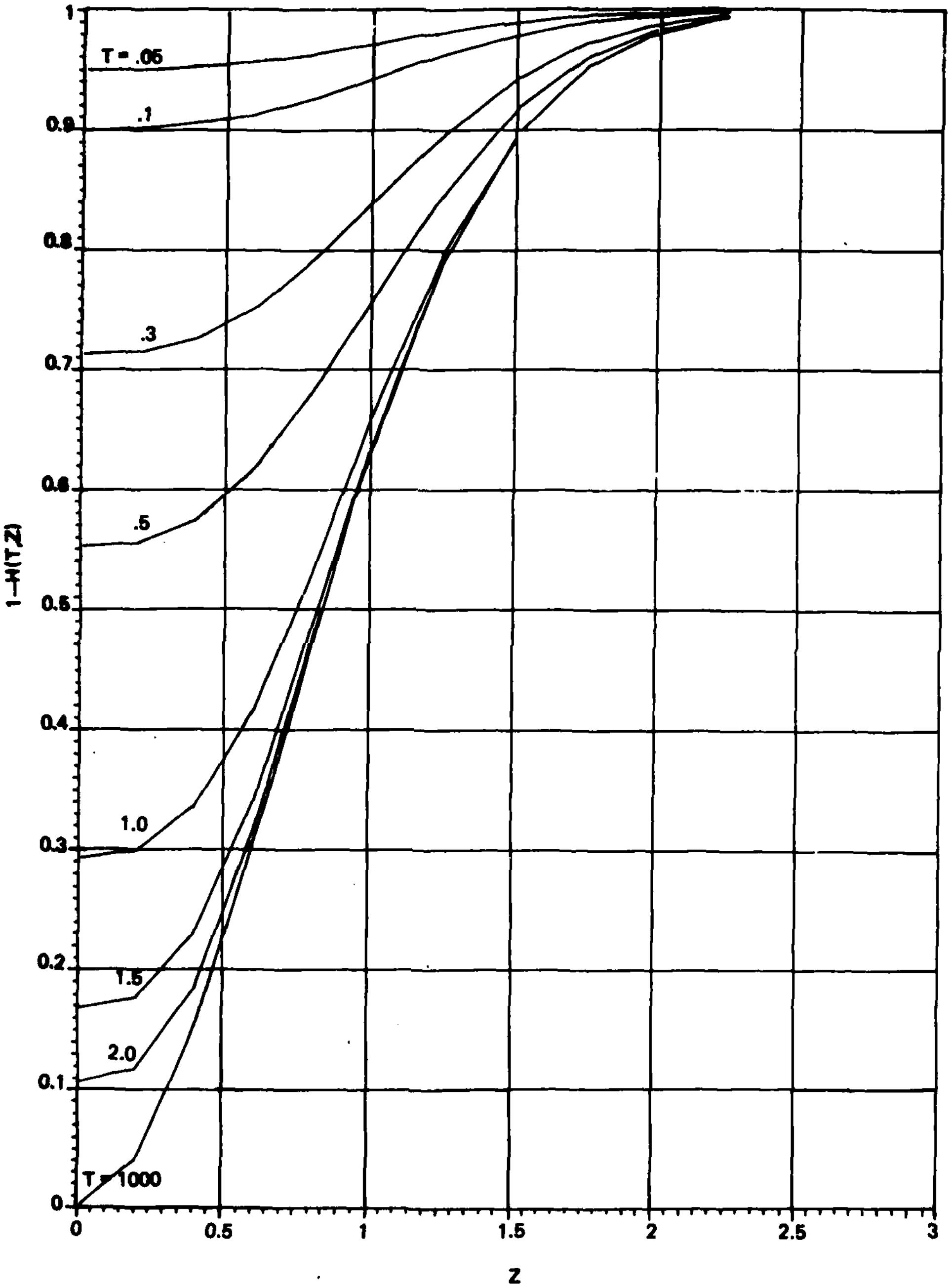

Figure 4. Fraction of data rejected vs. sum channel threshold.

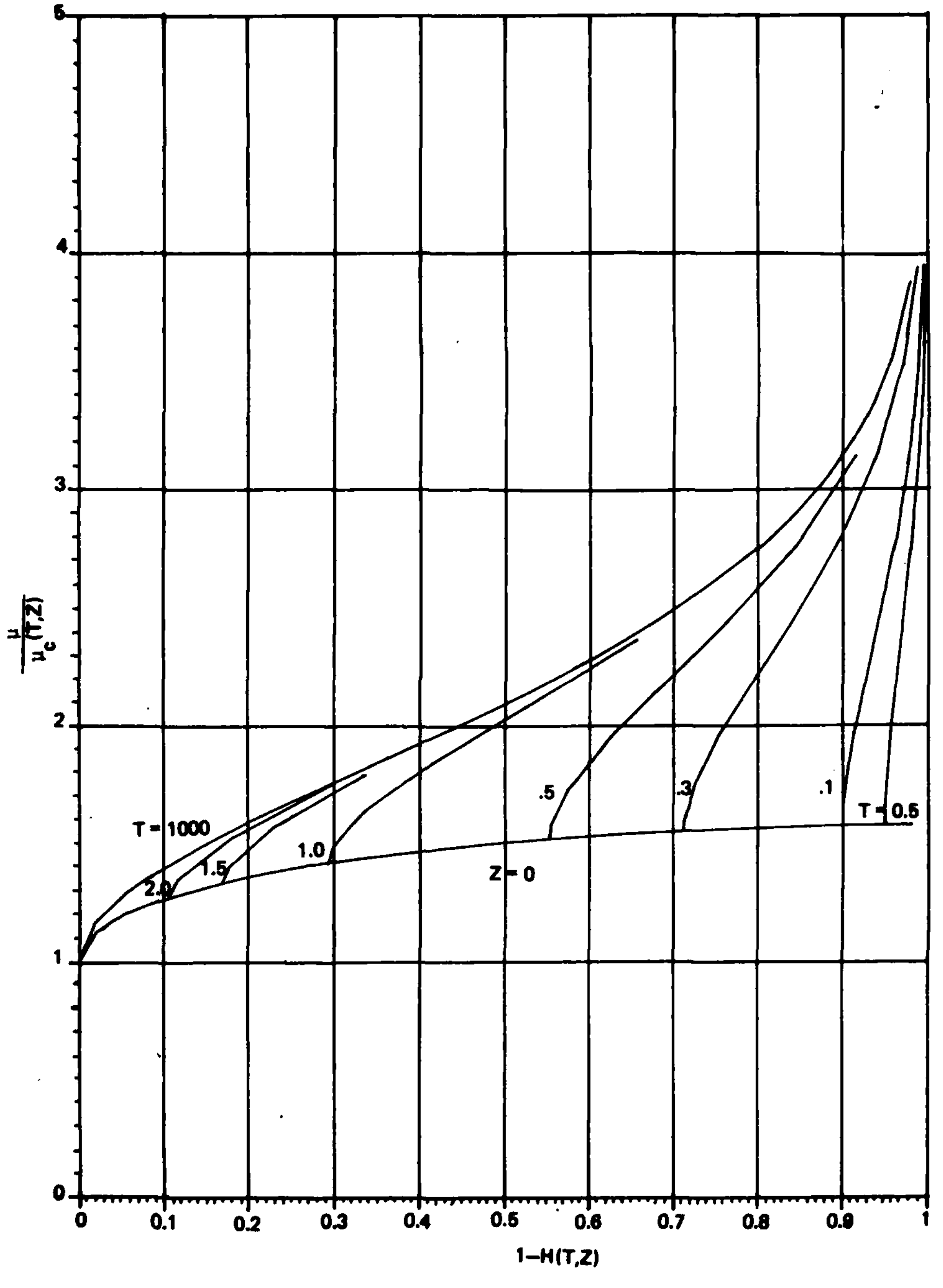

Figure 5. Glint reduction factor vs. fraction of data rejected.

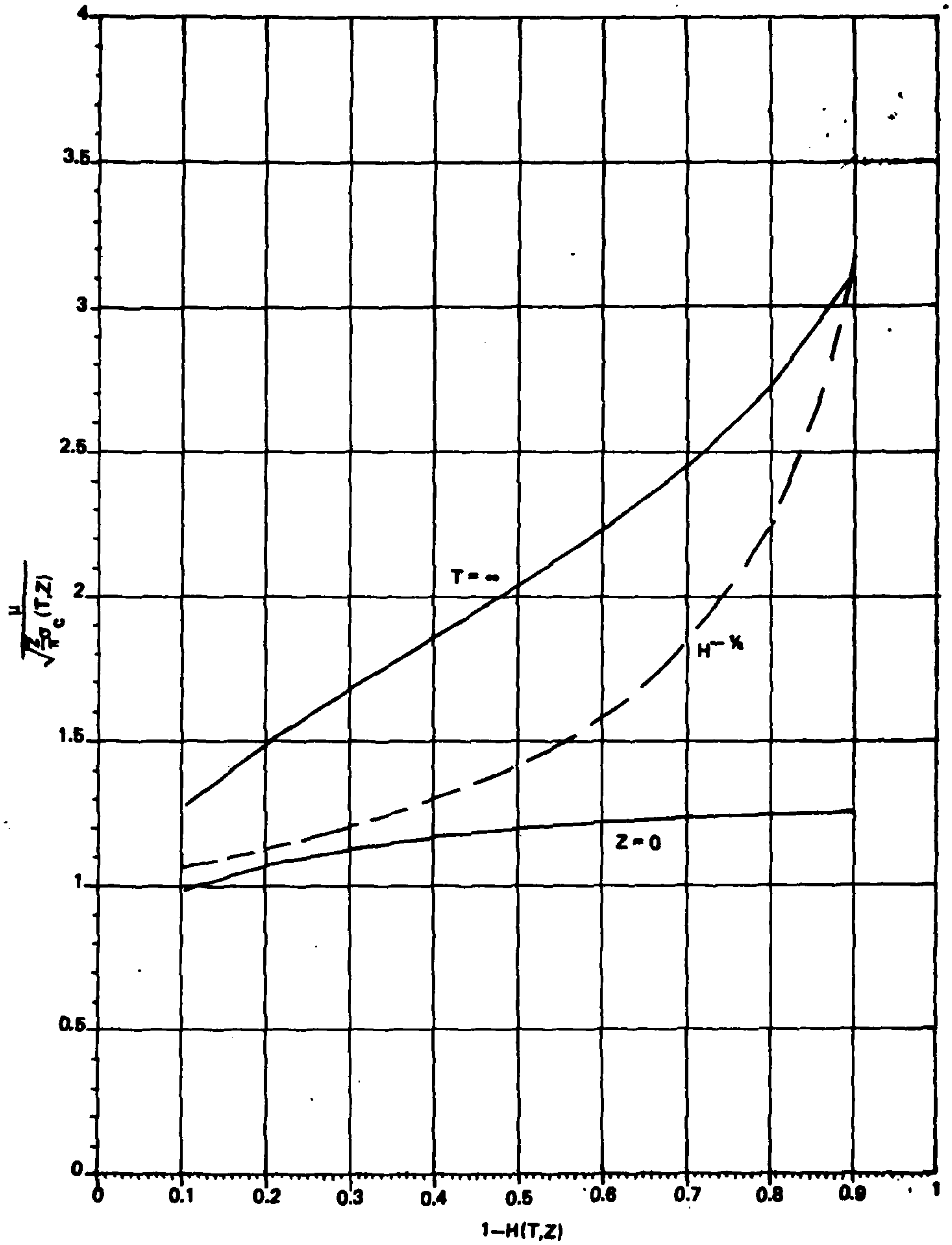

Figure 6. Glint reduction factor vs. fraction of data rejected.

APPENDIX

The preceding analysis gives the glint reduction when thresholds are applied. In order to find the actual glint before thresholding, the elements of the covariance matrix must be determined. These are given by (15), (16), and (17). Assume that $\overline{|A(\psi)|^2}$ is independent of ψ, i.e., assume a uniform distribution of scatterers across the target. Then

$$2b^2 = \overline{|A|^2} \int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} \left[\frac{1}{2} + \frac{1}{2} \cos\left(\frac{2\pi d}{\lambda} \sin\psi\right)\right] d\psi + \overline{|n_s|^2} \qquad \text{(A-1)}$$

$$2a^2 = \overline{|A|^2} \int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} \left[\frac{1}{2} - \frac{1}{2} \cos\left(\frac{2\pi d}{\lambda} \sin\psi\right)\right] d\psi + \overline{|n_D|^2} \qquad \text{(A-2)}$$

$$2\rho ab = \overline{|A|^2} \int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} \frac{1}{2} \sin\left(\frac{2\pi d}{\lambda} \sin\psi\right) d\psi \qquad \text{(A-3)}$$

We make the observation that the integral representation of the Bessel function of the first kind of integer order

$$J_n(z) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{i(z \sin\psi - n\psi)} d\psi \qquad \text{(A-4)}$$

is the $n^{\underline{th}}$ Fourier coefficient in the expansion of $e^{iz \sin\psi}$. Thus we obtain the Anjer-Weber representation

$$e^{i\frac{2\pi d}{\lambda} \sin\psi} = \sum_{n=-\infty}^{\infty} J_n\left(\frac{2\pi d}{\lambda}\right) e^{in\psi} \qquad \text{(A-5)}$$

Integrating over the target extent we have

$$\int_{\psi_o - \frac{\delta}{2}}^{\psi_o + \frac{\delta}{2}} e^{i\frac{2\pi d}{\lambda}\sin\psi} d\psi = \sum_{n=-\infty}^{\infty} J_n\left(\frac{2\pi d}{\lambda}\right) e^{in\psi_o} \int_{-\frac{\delta}{2}}^{+\frac{\delta}{2}} e^{in\phi} d\phi \tag{A-6}$$

The elements of the covariance matrix become

$$2b^2 = \frac{\overline{|A|^2}}{2}\delta \left[1 + J_o\left(\frac{2\pi d}{\lambda}\right) + 2\sum_{1}^{\infty} J_{2n}\left(\frac{2\pi d}{\lambda}\right)\cos 2n\,\psi_o \frac{\sin n\delta}{n\delta}\right] + \overline{|n_S|^2} \tag{A-7}$$

$$2a^2 = \frac{\overline{|A|^2}}{2}\delta \left[1 - J_o\left(\frac{2\pi d}{\lambda}\right) - 2\sum_{1}^{\infty} J_{2n}\left(\frac{2\pi d}{\lambda}\right)\cos 2n\psi \frac{\sin n\delta}{n\delta}\right] + \overline{|n_D|^2} \tag{A-8}$$

$$2\rho ab = \frac{\overline{|A|^2}}{2}\left[2\sum_{n=0}^{\infty} J_{2n+1}\left(\frac{2\pi d}{\lambda}\right)\sin(2n+1)\psi_o \frac{\sin(2n+1)\delta/2}{(2n+1)\delta/2}\right] \tag{A-9}$$

where we have employed

$$J_{-n}(z) = (-1)^n J_n(z) \tag{A-10}$$

Thus for specified antenna dimension, RF wavelength, target extent and average noise powers, the spread of the pdf of Re D/S before thresholding may be determined.

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